FLUID DYNAMICS AND AERODYNAMICS

MCE 412

INTRODUCTION TO FLUID DYNAMICS

Fluid Mechanics is that branch of applied science that studies the action of forces on fluids and the effects these forces can produce using fluids.

Like mechanics of solids, fluid mechanics is logically divided into static and dynamics. In statics, forces acting on fluids at rest were studied. Fluid statics is also called hydrostatics, when we consider only liquid fluids. In dynamics, forces acting on the moving fluid are studied.

In the study of statics, fluid weight (acting vertically downward) and static thrust (acting on the faces of the fluid) were the significant properties of interest; but when a fluid begins to move; it obeys the same basic laws of motion as a solid body. That means a force must have been applied to generate the resultant acceleration, and the only way of applying a force to the element of fluid is by modifying the pressures extended on the fluid by the surrounding liquids.

Fluid dynamics is divided into four divisions

- Hydro-kinematics: deals separately with the motion of liquid fluids (without considering the forces responsible for the motion)
- Hydro-kinetics-deals separately with the forces responsible for the motion of liquid fluids
- Gas dynamics-deals with the motion of compressible fluids (gases and vapours)
- Aerodynamics- deals with the interaction of the atmosphere with a solid body in motion.

At the level of studying fluid dynamics, the subject should be structured in such a way; that both the motion of liquids and gases are treated simultaneously, using the same basic method of analysis.

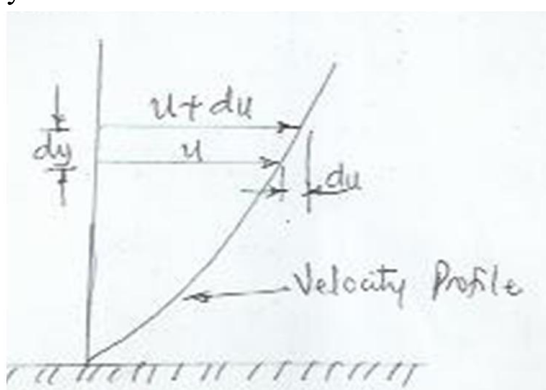
For a fundamental treatment, it is necessary to examine the forces and energies involved in fluid motion. Thus the significant properties of interest are velocity and acceleration.

The branch of fluid dynamics where velocity and acceleration are the significant properties of interest is called “kinematics of fluid flow”. The fundamental equations for studying the kinematics of fluid flow are developed from the basic laws governing the motion of solid body. These include:

(1) The conservation of mass; from which the continuity equation is developed
(2) The conservation of energy from which the energy equations are derived
(3) The conservation of momentum; from which equations evaluating dynamics forces exerted by the flowing fluids are established.

Properties of Fluids

- ***Density or mass Density***: Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by symbol e(rho). The unit of mass density in SI in kg/m^3 to its volume. It is denoted by the symbol w.
- ***Specific volume***: Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume. It is expressed as m^3/kg. It is commonly applied to gasses.
- ***Specific gravity***: Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid. It is a dimensionless quantity and is denoted by the symbol S.
- ***Viscosity:*** Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluids. When two layers of a fluid at distance 'dy' apart, moves over each other at different velocities, say u + du as shown below, the viscosity together with relative velocity causes a shear stress acting between the fluid layers.



$\longrightarrow$ U

Velocity variation near a solid boundary

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. it is denoted by symbol τ (Tau)

Mathematically, $\tau \propto \dfrac{du}{dy}$

Or $\tau = \mu \dfrac{dy}{dy}$

Types of Fluids

Fluids may be classified into the following types:

- Ideal fluid: A fluid, which is incompressible and having no viscosity, is known as an ideal fluid. An ideal fluid is only an imaginary fluid . All existing fluids have some viscosity.
- Real fluid: A fluid which possesses viscosity is known as real fluid. All the fluids, in actual practice are real fluids.
- Newtonian Fluids: A real fluid in which the shear stress is directly proportional to the rate of shear strain (or velocity gradient) is known as Newtonian fluid.
- Non-Newtonian fluid: A real fluid in which the shear stress is not proportional to the rate of shear strain (or velocity gradient) is known as a Non-Newtonian fluid.
- Ideal plastic fluid. A fluid in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient is known as an ideal plastic fluid.

Methods for studying or describing fluid flow patterns/movement of flow

There are two methods for studying the movement of flow. One is a method which follows arbitrary particle with its kaleidoscopic changes in velocity and acceleration. This is called the Lagrangian method. The other is a method by which, rather than following any particular fluid particle, changes in velocity and pressure are studied at fixed positions in space x,y,z and at time t. This method is called the Eulerian method. Nowadays the latter method is more common and effective in most cases.

The Lagrangian method

The langrangian method is best illustrated using the solid body mechanics. In the solid body mechanics, we are used to describing the motion of a body in terms of its position, versus time.

The langrangian method is commonly used for studying the kinematics of solids where it is convenient to identify a discrete, particle e.g. Centre of mass of spring-mass system and to determine the subsequent history of its movement in time. The langrangian method when applied to a fluid body as a continuum of particle the method becomes extremely cumbersome. Hence, it is necessary to adopt a different method.

In a fluid body it is necessary to observe the motion of the fluid particle as they pass a given location in the flow field. Unlike a solid body, when a fluid body moves from one position to the next, it is usually deforms continuously. Therefore, in order to describe completely the motion of a fluid body, it is necessary to account for its deformation as well as its translation and rotation.

In addition, it is often necessary to determine the velocity and pressure distribution about a fluid body with given size and shape. Information about the flow is required at specified location in the flow field. The method of analysis that seek to analyse the motion of fluid particles (as a body) as the fluid body pass a given location is called Eulerian method or the control volume method.

The Eulerian Method

The Eulerian method, generally called the “Control Volume” method enables one to fix attention at discrete point without regards to the identity of the individual particles of fluid occupying these points at a givens instant. In using the Eulerian method, the observer notes the flow characteristic in the vicinity of a fixed point as particles pass by.

The description of the entire flow field is essentially an instantaneous picture of the velocities and accelerations of every particles as a body.

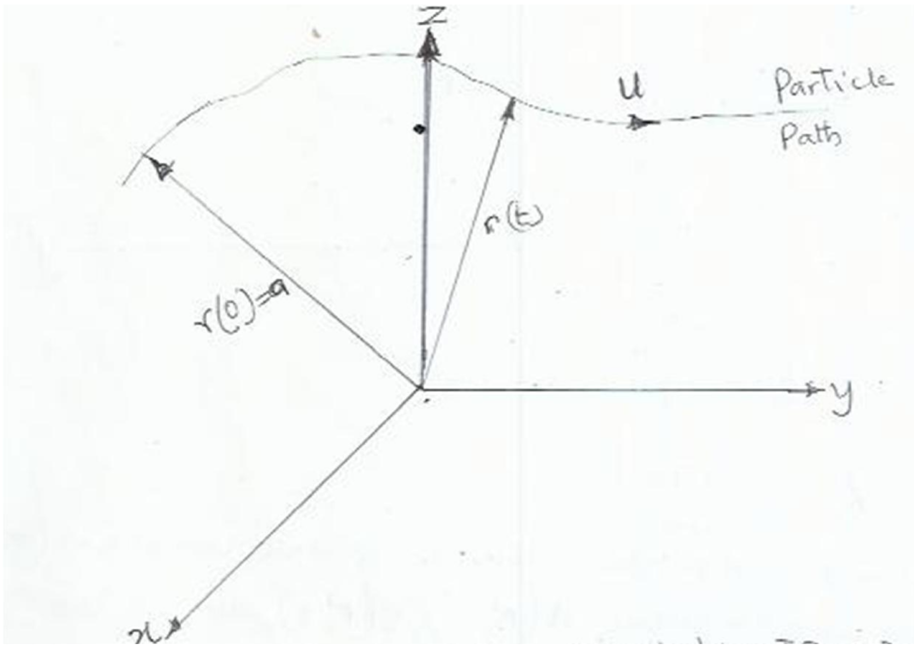
The main difference between the Lagrangian and the Eulerian method lie in the fact:

(1) In the langrangian approach, the coordinates of the particles are represented as functions of time and hence they are dependent variables.
(2) In the Eulerian approach, the particle velocities as a body at various points are given as functions of time and hence they are independent variables.

To apply the Eulerian method, first, a relationship between the two basic methods (the langrangian and the Eulerian methods) would have to be developed.

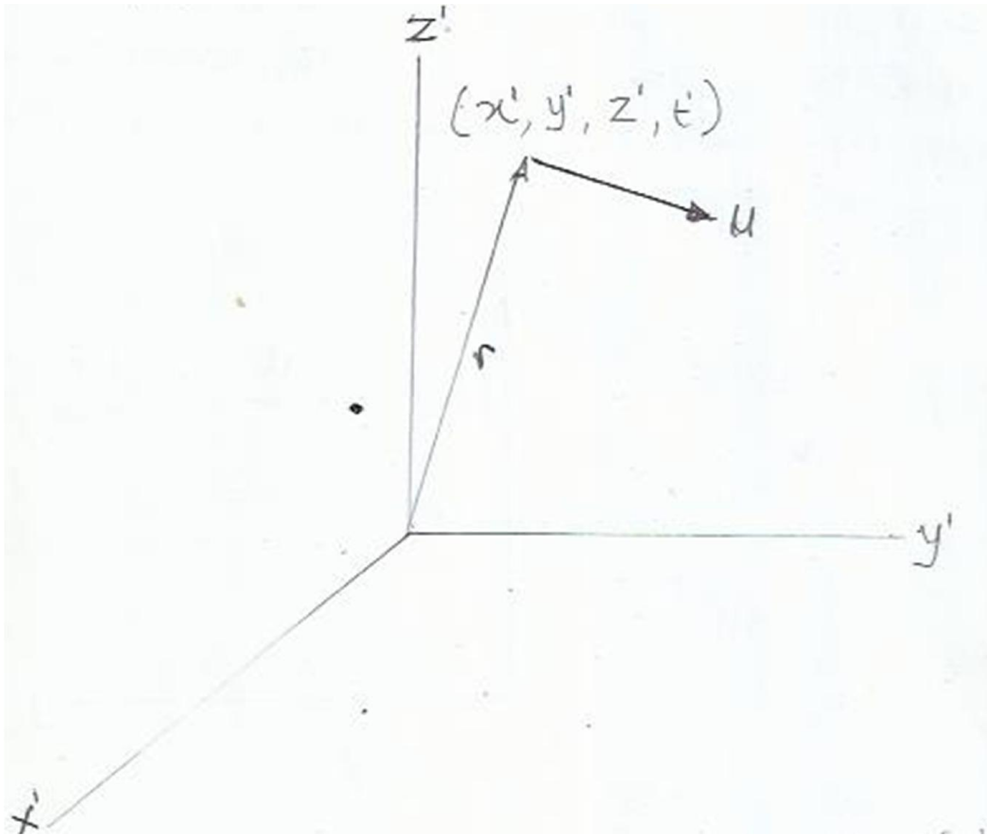
Second, since the laws governing the motion of solid body also apply to fluid body, these laws have to be adopted to the motion of fluid.

In Langrangian description, one essentially follows the history of individual fluid particles (see the figure below). Consequently, the two independent variables are taken as time and a label for fluid particles. The label can conveniently be taken as the position vector a of the particle at some reference time $t = 0$. In this description, any flow variable F is expressed as $F(a,t)$. In particular , the position vector is written as $r = r(a,t)$, which represents the location at t of a particle whose position was a at $t = 0$



Particle – Langrangian description. Independent variables (a,t); dependent variables: r (a,t), u= $(\partial r / \partial t)_a$ $e = \rho(a,t)$ and so on.

In the Eulerian description, one concentrates on what happens at a spatial point r^1, so that the independent variables are taken as r^1 and t^1. (Here the primes are meant to distinguish Langrangian dependent variables from Eulerian independent variables). Flow variables are written, for example as $F(r^1,t^1)$



Field-Eurlerian description. Independent variables (x^1, y^1, z^1, t^1) dependent variable : $u(r^1, t^1), \rho(r^1, t)$ and so on.

The velocity and acceleration of a fluid particle in the Lagrangian description are simply the partial time derivatives as the particle identity is kept constant during the differentiation.

$u = \partial r / \partial t$, acceleration $a = \partial u / \partial t = \partial^2 r / \partial t^2$

Basic Scientific Laws used in the Analysis of fluid flow

As noted earlier in the introduction, the basic laws are related to the conservation of: mass, momentum and energy.

(i) **Law of conservation of Mass**: This law when applied to a control volume states that the net mass flow through the volume will equal the mass stored or removed from the volume. Under conditions of steady flow this will mean that the mass leaving the control volume should be equal to the mass entering the volume. The determination of flow velocity for a specified mass flow rate and flow area is based on the continuity equation derived on the basis of this law.

(ii) **Newton's law of motion**: These are basic to any force analysis under various conditions of flow. The resultant force is calculated using the condition that it equals the rate of change of momentum. The reaction on surfaces are calculated on the basis of these laws. Momentum equation for flow is derived based on these laws.

(iii) **Law of Conservation of Energy:** Considering a control volume, the law can be stated as “the energy flow into the volume will equal the energy flow out of the volume under “steady conditions”. This also leads to the situation that the total energy of a fluid element in a steady flow field is conserved. This is the basis for the derivation of Euler and Bernoulli equations for fluid flow.

Note: A control volume by definition is a volume fixed with respect to a coordinate system in a flow field.

Fundamental Definition of Fluid Particles

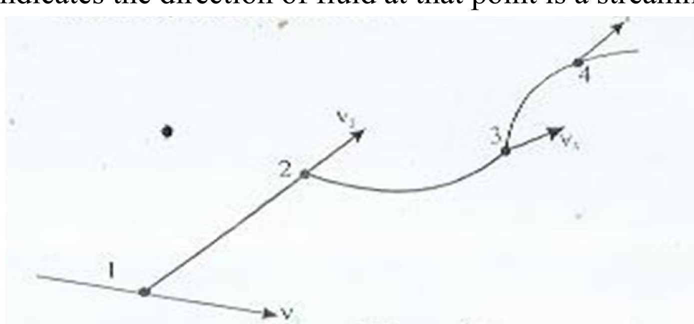
To describe a fluid particle in a flow, there must exist a flow field. This is a region in which the flow is defined in terms of space and time co-ordinate.

Generally, a fluid consists of a large number of individual particles moving in the general direction of flow, but usually not parallel to each other.

The velocity of any particle is a vector quantity having magnitude and direction which may vary from movement to movement. Thus, we have the following description of fluid particle and the flow fluid.

(i) Path Line: This is the path followed by a particle in the flow fluid. At any given time, the positions of the successive particles can be joined by a curve.

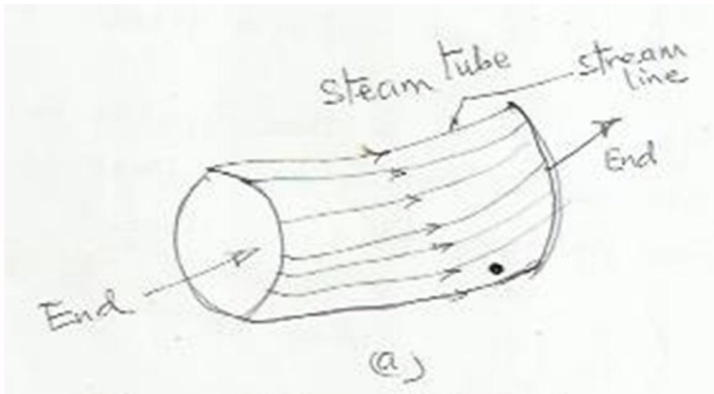
(ii) Streamline: This is a curve joining the positions of successive particles in a flow field. The curve is tangential to the direction of motion of the particle at that instant, and it is ordinary in three dimensions. In other words, the curve where the tangent at each point indicates the direction of fluid at that point is a streamline.



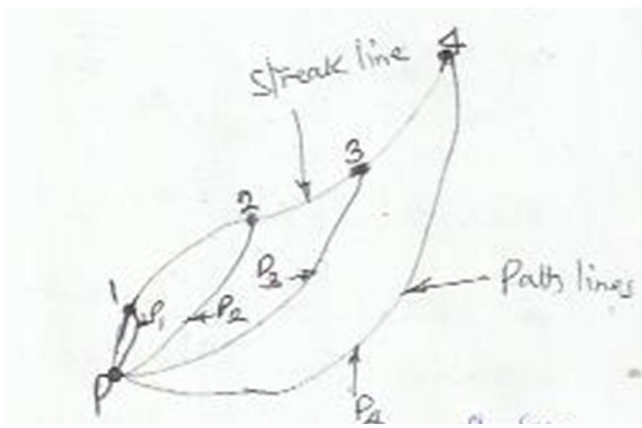
A stream line

In a flow field, the velocity of a particle at any point on a stream line is tangential to it, so there can be no flow across a stream line. A stream line therefore shows the direction of the velocity vector at any instant.

(iii) Stream tube: This is an imaginary flow passage formed by a number of adjacent stream lines. This tube is forced by drawing through every point on the circumference of a small area in the flow field.



Stream tubes



Path lines and Streak lines

If the cross-sectional area of the stream tube is small enough for the velocity to be considered constant, the flow passage is called a "stream filament".

(iv) Streak line: This is a line of fluid particles, all of which passed through the same point in the flow field at a previous time.

In experimental work, streak lines are obtained by injecting dye, smoke or particles into the moving fluid and observing the subsequent flow pattern.

Nature or types of fluid flow

There are many ways to classify fluid flow problems, the following are some general categories:

- **Steady and Unsteady flow**

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc at a point do not change with time. Thus, for steady flow,

$$\left(\frac{\partial v}{\partial t}\right)_{x_o, y_o, z_o} = o, \left(\frac{\partial p}{\partial t}\right)_{x_o, y_o, z_o} = o, \left(\frac{\partial e}{\partial t}\right)_{x_o, y_o, z_o} = o$$

Where $(x_{o,} y_o, z_o)$ is a fixed point in fluid field

Unsteady flow is that type of flow, is that type of flow in which the velocity, pressure and density at a point changes with respect to time. For unsteady flow,

$$\left(\frac{\partial v}{\partial t}\right)_{x_o, y_o, z_o} \neq o, \left(\frac{\partial p}{\partial t}\right)_{x_o, y_o, z_o} \neq o \text{ e.t.c}$$

- **Uniform and Non-Uniform flows**

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e. length of direction of the flow). For uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t = cons\tan t} = o$$

Where $\partial v =$ change of velocity

$\partial s =$ Length of flow in the direction S

This statement implies that other fluid variables do not change with distance. Thus,

$$\frac{dv}{ds} = o, \frac{\partial p}{\partial s} = o, \frac{\partial \rho}{\partial s} = o$$

Also, in a uniform flow, the velocity (v) is identically same at all points at a given instant. For instance, flow of liquid under pressure through pipelines of constant diameter is uniform flow whether the flow is steady or unsteady.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus for non-uniform flow.

$$\left(\frac{\partial v}{\partial s}\right)_{t = cons\tan t} \neq o$$

- **Laminar and Turbulent flows**

Laminar flow is derived as that type of flow in which the fluid particles more along well-defined paths or streamline and all the streamlines are straight and parallel. Thus the particles moves in laminars or layers sliding smoothly over the adjacent layer. This type of flow is also called streamline flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles moves in a zig-zig or disorderly manner, occupying relative positions in successive cross-section. To determine whether a flow is laminar or turbulent, Osborne Reynolds Number (Re) as

$$\text{Re} = \frac{\rho v d}{\mu} = \frac{v d}{v} = v\frac{(4R)}{v}$$

Where, $\rho =$ Density of the fluid

$v = velocity$

d = A typical dimension of the flow passage (for a pipe, d = diameter of the pipe)

$\mu =$ Dynamic velocity of the fluid

In particular, whenever the velocity is the critical velocity Vc, Re = $\frac{V_c d}{v}$ is called critical Reynolds number.

- **Compressible and Incompressible**

In general, liquid is called an incompressible fluid and gas as a compressible fluid. A flow is classified as being compressible, depending on the level of variation of density during flow. Compressible flow is that type of flow in which the density of the fluid changes from point to point.

Incompressible flow is that type of flow in which the density is constant for the fluid flow.

- **Internal and External flow**

A fluid flow is classified as being internal or external, depending on whether the fluid flows is in a confined space or over a surface. The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is <u>external flow</u>. The flow in a pipe or duct is internal flow if the fluid is completely bounded by a solid surface.

- **Viscous and Inviscid Regions of Flow**

When two fluid layers moves relative to each other, a frictional force develops between them and the slower layer tries to slow down the faster layer. This internal resistance to flow is qualified by the property viscosity which is a measure of internal stickiness of the fluid. Flows in which the frictional effects are significant are called viscous flows.

- **Rotational or Irrotational:**

Rotational flow is that type of flow in which the fluid particles while flowing along streamlines, also rotate about their own axis. If the fluid particles while flowing along streamlines, do not rotate about their own axis that type of flow is called irrotational flow.

Three-dimensional, two-dimensional and one-dimensional flow.

All general flows such as a ball flying in the air and a flow around a moving automobile have velocity components in x, y and z direction. In other word, 3 Dimensional type of flow is that type of flow in which the velocity is a function of time and type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (u,y and z). Expressing the velocity components in the x,y and z axial direction u,v and w, then

$$u = U(x, y, z),\ v = V(x, y, z),\ w = W(x, y, z)$$

Consider water running between two parallel plates cross-cut vertically to the plates and parallel to the flow. If the flow states are on all planes parallel to the cut plane, the flow is called a two-dimensional flow since it can be described by two coordinates x and y. expressing the velocity components in the x and y directions as u and v respectively, then

$$u = U(x, y, t),\ v = V(x, y, t),\ w = 0$$

A flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. for one-dimensional flow

$$u = U(x),\ v = 0,\ w = 0$$

Laws Governing the motion of bodies to fluid motion

Fluid mechanics is based on the conservation laws of mass, momentum and energy. Historically, the conservation laws are first applied to a fixed quantity of matter called a <u>closed system</u> or just a system, and then extended to region in space called <u>control volumes.</u> The conservation relations are also called balance equations since any conserved quantity must balance during a process.

- **Conservation of Mass**

The conservation of mass relation for a closed system undergoing a change is expressed as m_{sys} = constant or $\frac{dm_{sys}}{dt} = 0$, which is the statement that mass of the system remains constant during a process. For a control volume (CV), mass balance is expressed in rate form as:

conservation of mass: $\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{cv}}{dt}$

Where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the total rates of mass flow into and out of the control volume, respectively, and $\frac{dm_{cv}}{dt}$ is the rate of change of mass within the control volume boundaries.

- **Conservation of Momentum**

The product of the mass and velocity of a body is called the linear momentum or just the momentum of the body, and the momentum of a rigid body of mass in moving with a velocity $\vec{V}$ *is* $m\vec{V}$. Newton's second law states that the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.

Therefore, the momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such systems is conserved. This is known as the conservation of momentum principle. In fluid mechanics, Newton's second law is usually referred to as the linear momentum equation, together with angular momentum equation.

- **Conservation of Energy**

Energy can be transferred to or from a closed system by heat or work, and the conservation of energy principle requires that the net energy transfer to or from a system during a process be equal to the change in the energy content of the system. Control volumes involve energy transfer via mass flow also, and conservation of energy principle, also called the energy balance, is expressed as

Conservation of energy: $\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{cv}}{dt}$

Without $\dot{E}_{in} - \dot{E}_{out}$ are the total rates of energy transfer into and out of the control volume, respectively, and $\frac{dE_{cv}}{dt}$ is the rate of change of energy within the control volume boundaries.

The law of conservation of energy states that the time rate of change of total energy possessed by a system of particles is equal to the rate of addition of heat energy to the system, less the rate of work done by the system

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

- **Conservation of Mass Principle**

The conservation of mass principle for a control volume can be expressed as: The net mass transfer to or from a control volume during a time interval Δt is equal to net change (increase or decrease) of the total mass within the control volume during Δt. That is,

$$\begin{pmatrix} Total\ mass\ entering\ the \\ the\ CV\ during\ \Delta t \end{pmatrix} - \begin{pmatrix} Total\ mass\ leaving \\ the\ CV\ during\ \Delta t \end{pmatrix} = \begin{pmatrix} Net\ change\ of\ mass\ within \\ the\ CV\ during\ \Delta t \end{pmatrix} \text{or}$$

$$m_{in} - m_{out} = \Delta m_{cv}$$

Where $\Delta m_{cv} = m_{final} - m_{initial}$ is the change in the mass of the control volume during the process. It can also be expressed in rate form as

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{cv}}{dt} \ (kg/s)$$

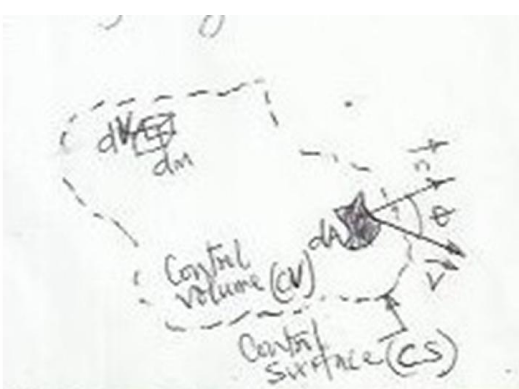
Where $\dot{m}_{in} - \dot{m}_{out}$ are the total rates of mass flow into and out of the control volume, and dm_{cv}/dt is the rate of change of mass within the control volume boundaries.

Consider a control volume of arbitrary shape in the figure below, the mass of a differential volume dV within the control volume is $dm = \ell dV$. The total mass within the control volume at any instant in time t is determined by integration to be

Total mass within the CV: $M_{cv} = \int \rho \partial v$

Then the time rate of change of the amount of mass within the control volume is expressed as

Rate of change of mass within the CV $\frac{{}^{dm}cr}{dt} = \frac{d}{dt}\int_{cv} \rho dv$



For a special case of no mass crossing the control surface (i.e., the control volume is a closed system), the conservation of mass principle reduces the $dm_{cv}/dt = 0$ This is valid whether the control volume is fixed, moving, or deforming.

Now consider mass flow into or out of the control volume through a differential area dA on the control surface of a fixed control volume. Let $\vec{n}$ be the outward unit vector of dA normal to dA and $\vec{V}$ be the flow velocity at dA relative to a fixed coordinate system as shown above. In general, the velocity may cross dA at an angle θ off the normal of dA, and the mass flow rate is proportional to the normal component of velocity $\vec{V}_n = \vec{V}\cos\theta$ ranging from a maximum outflow of $\vec{V}$ for $\theta = 0$(flow is normal to dA) to a minimum of zero for $\theta = 90^o$(flow is tangent to dA) to a maximum inflow of $\vec{V}$ for $\theta = 180^o$ (flow is normal to dA) but in the opposite direction). Making use of the concept of dot product of two vectors, the magnitude of the normal component of velocity is

Normal component of velocity: $V_n = V\cos\theta = \vec{v}.\vec{n}$

The mass flow rate through dA is proportional to the fluid density e, normal velocity V_n, and the flow area dA, and is expressed as

Differential mass flow rate: $\partial m = \ell v_n dA = \ell(v\cos\theta)dA = \ell(\vec{v}.\vec{n})dA$

The net flow rate into or out of the control volume through the entire control surface is obtained by integrating ∂m over the entire control surface,

Net mass flow rate: $m_{net} = \int_{cs} \partial m = \int_{cs} \rho v_n dA = \int_{cs} \rho(\vec{v}.\vec{n})dA$

Note $V_n = \vec{v}.\vec{n} = v\cos\theta$ *is positive for* $\theta ,< 90^0$ (*outflow*) *and negative for* $\theta > 90^0$ (*inflow*).. Therefore, the direction of flow is automatically accounted for, and the surface integral in above equation directly gives the net mass flow rate. A positive value for m_{net} indicates a net outflow of mass and a negative value indicates a net inflow of mass.

General conservation of mass: $\frac{d}{dt}\int_{cv} \rho dv + \int_{cs} \rho(\vec{v}.\vec{n})\, dA = 0$

It states that the time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

Conservation of Momentum Principle

Newton’s laws are relations between motion of bodies and the forces acting on them. For a rigid body of mass m, Newton’s law is expressed as :

$$\vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = d\frac{(mv)}{dt}$$

Where $\vec{F}$ is the net force acting on the body and a is the acceleration of the body *under the influence of* $\vec{F}$.

The momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such a system is conserved. This is as the conservation of momentum principle.

In fluid mechanics, however, the net force acting on a system is typically not zero, and we prefer to work with the linear momentum equation rather than the conservation of momentum principle.

The counterpart of Newton's second law for rotating bodies is expressed as $\vec{M} = I\vec{\alpha}$, where $\vec{M}$ is the net moment or torque applied on the body, I is the moment of Inertia of the body about the axis of rotation and $\vec{\alpha}$ is the angular acceleration. It can also be expressed in terms of the rate of change of angular momentum $\frac{d\vec{H}}{dt}$ as

Angular momentum equation $\vec{M} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = d\frac{(I\vec{\omega})}{at} = \frac{d\vec{H}}{dt}$

Where ω is the angular velocity. For a rigid body rotating about a fixed x-axis, the angular momentum equation is written in scalar form as

Angular momentum about x-axis $M_x = I_x\frac{d\omega_s}{dt} = \frac{dH_x}{dt}$

Forces acting on a Control Volume

The forces acting on a control volume consist of body forces that act throughout the entire body of the control volume (such as gravity, electric and magnetic forces) and surface forces that act on the control surface (such as pressure and viscous forces and reaction forces at points of contact.

A control volume can be selected as any arbitrary region in space through which fluid flows, and its bounding control surface can be fixed, moving, and even deforming during flow.

In control volume analysis, the sum of all forces acting ion the control volume at a particular instant in time is represented by $\sum\vec{F}$ and is expressed as

Total force acting on control volume $\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface}$

Newton's second law for a system of mass m subjected to net force $\sum \vec{F}$ is

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

Where $m\vec{v}$ is the linear momentum of the system.

More generally, Newton's second law may be expressed as

$$\sum \vec{F} = \frac{d}{dt}\int_{sys} \rho\vec{v}dv$$

Where $\rho\vec{v}dv$ is the momentum of a differential elemental dV which has mass $\partial m = \rho dv$

The general form of linear momentum equation that applies to fixed, moving or deforming control volumes is

$$\sum \vec{F} = \frac{d}{dt}\int_{cv} \rho\vec{v}dv + \int_{cs} \rho\vec{v}(\vec{v}.\vec{n})\, dA$$

Which is word implies

$$\begin{pmatrix} \text{The sum of all} \\ \text{external forces} \\ \text{acting on a Cv} \end{pmatrix} = \begin{pmatrix} \text{That time rate of change} \\ \text{of linear momentum of the} \\ \text{contents of the CV} \end{pmatrix} + \begin{pmatrix} \text{The net flow rate of} \\ \text{linear momentum out of the} \\ \text{control surface by mass flow} \end{pmatrix}$$

For a fixed control volume (no motion or deformation of control volume) $V_r = V$ and the linear momentum equation becomes

$$\text{Fixed CV} = \sum \vec{F} = \frac{d}{dt}\int_{cv} \rho\vec{v}dv + \int_{cs} \rho\vec{v}(\vec{v}.\vec{n})dA$$

During steady flow: $\sum \vec{F} = \int_{cs} \rho\vec{v}(\vec{v}_r.\vec{n})dA$

$\vec{V}$ = fluid velocity as viewed form an inertial reference frame

The product $\rho(\vec{v}_r.\vec{n})dA$ = the mass flow rate through area element dA into or out of the control volume.

ENERGY EQUATION

For a system of fluid particles, the law of conservation of energy states that the total energy E of the system increases from state 1 to state 2 by an amount equal to the total heat Q added to the system of particles less the work done W by the system of fluid particles;

$$E_2 - E_1 = Q - W$$

E is the total energy possessed by the system in a given state and thereby includes the kinetic and potential energy of the entire system mass, the internal energy associated with random motion of the molecules comprising the system, and other forms of storable energy.

The above equation in differential form becomes

$$dE = dQ - dW$$

Applying law of conservation of energy to a control volume in a fluid, in which mass is entering and leaving across the surface bounding the control volume.

$$\frac{dE}{dt}\Big|_{system} = \frac{dE}{dt}\Big|_{cv} + \int e\ell v_n dA$$

$$e = energy\ \ per\ unit\ mass$$

$$\frac{d}{dt}(Q - W) = \frac{dE}{dt}\Big|_{cv} + \int e\rho v_n d_A$$

If the system possesses only internal, kinetic the potential energies

$$E = U + K.E + P.E\ or\ U + \frac{v^2}{2} + gz$$

$$\therefore \frac{d}{dt}(Q - W) = \frac{dE}{dt}\Bigg|_{cv} + \int_{cs}\left(U + \frac{v^2}{2} + gz\right)\rho v_n dA$$

Euler's Equation of Motion

This is equation of motion in which the force due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a streamline as:

Consider a streamline in which flow is taking place in s-direction as shown below. Consider a cylindrical element of cross-section dA and length ds. The force acting on the cylindrical element are:

1. Pressure force pdA in the direction of flow

2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds \right) dA$ opposite to the direction of flow
3. Weight of element $egdAds$

Let is the angle between the direction of flow and the line of action of the weight of element

The resultant force on the fluid element in the direction is S must be equal to the mass of fluid element x acceleration in the directions.

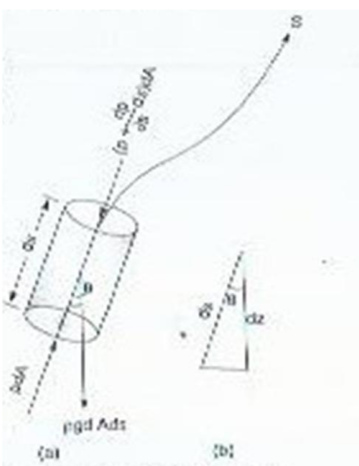
$$\therefore \text{ pdA - } \left(p + \frac{\partial p}{\partial s} ds \right) dA - \rho g dAds \cos\theta$$

$$= pdAds \times a_s$$

Now $a_s = \frac{dv}{dt}$,

Where v is a function of s and t

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial V}{dt} = 0$$



Forces on a fluid element

If the flow is steady, $\frac{dV}{dt} = 0$

$$\therefore as = \frac{VdV}{ds}$$

Substituting the value of a_s in the equation above we get mass

$$-\frac{\partial p}{ds}dsdA-\ell gdAds\cos\theta=\rho dAds\times\frac{VdV}{dS}$$

Dividing by ρdsdA,

$$-\frac{\partial p}{\rho ds}-g\cos\theta=\frac{VdV}{dS}$$

$$or\ \frac{\partial p}{\rho ds}+g\cos\theta+\frac{Vdv}{dS}=0$$

But from fig(b) we have $\cos\theta=\frac{dz}{ds}$

$$\therefore\frac{1}{\ell}\frac{\partial p}{\partial S}+g\frac{dz}{ds}+\frac{Vdv}{\partial s}=0\ \ or\ \frac{dp}{\ell}+gdz+Vdv=0$$

$$\frac{\partial p}{\ell}+gdz+vdv=0\ \left[Euler's\ equation\ of\ motion\right]$$

Bernoullis equation from Eulers Equation

Bernoulli equation is obtained by integrating the Eurler's equation of motion as

$$\int\frac{dp}{\ell}+\int gdz+\int vdv=cons\tan t$$

If the flow is incompressible, ρ is constant and

$$\therefore\frac{P}{\ell}+gz+\frac{v^2}{2}=cons\tan t$$

$$or\ \frac{P}{\ell g}+Z+\frac{v^2}{2g}=cons\tan t$$

$$or\ \frac{P}{\ell g}+\frac{v^2}{2g}+z=cons\tan t\ \left[Bernoulli's\ Equation\right]$$

Where $\frac{P}{\rho g}$ pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ Kinetic energy per unit weight or kinetic lead.

Z = potential energy per unit weight or potential head.

Assignment

(1) What is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0mls.find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

(2) A pipe, through which water is flowing is having diameter 20am and 10am at the cross=-section 1 and 2 respectively. The velocity of water at section 1 is given as 4.0m/s. Find the velocity head at sections 1 and 2 and also the rate of discharge.